STAT 461/661
Practice Final 1
December 19, 2018

Name _____ Student ID #_____

Problem	Points	S C O R E
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Consider a random variable X with distribution

$$\mathbf{P}(X = -1) = 0.4, \quad \mathbf{P}(X = 0) = 0.4, \quad \mathbf{P}(X = 1) = 0.2$$

Find its expectation and the standard deviation.

2. (10 points) Find a CDF for a random variable X with density

$$f(x) = \frac{1}{4}x^3, \ 0 \leq x \leq 2.$$

3. (10 points) Take two events A and B such that

$$\mathbf{P}(A \mid B) = 0.3, \quad \mathbf{P}(B \mid A) = 0.2, \quad \mathbf{P}(A \cup B) = 0.9.$$

Find the probability of A.

4. (10 points) Consider a plane with $N = 300$ people. Assume half of the plane are chicken-lovers, who choose chicken with probability 75%, and the other half is pasta-lovers, who choose chicken with probability only 30%. All choices are independent. Find the quantity of chicken you need to store to satisfy chicken-lovers with probablity 99%.

5. (10 points) There are $N_1 = 1000$ high-risk clients, each of whom can have an accident with probability $p_1 = 0.06\%$, and $N_2 = 1000$ low-risk clients, each of whom can have an accident with probability $p_2 = 0.03\%$. All accidents occur independently. Given that the number of accidents is less than 3, find the probability that there are no accidents.

6. (10 points) Take two independent random variables $X \sim \mathrm{Exp}(2)$ and $Y \sim \mathcal{N}(-3, 10)$. For the random variable $Z = 1 + 2X - 3Y$, find $\mathbf{E}Z$ and $\operatorname{Var} Z$.

7. (10 points) For a random variable $X \sim \text{Uni}[1, 2]$, find the density of X^2.

8. (10 points) The joint density of two random variables X and Y is given by

$$p(x, y) = c(x^2 + y), \quad 0 \leq x \leq y, \quad 0 \leq y \leq 1,$$

and $p(x, y) = 0$ otherwise, where $c > 0$ is a constant. Find c.

9. (10 points) There are 50% low-risk, 30% medium-risk, and 20% high-risk clients. Each year, a high-risk client has an accident with probability 30%, a medium-risk client has an accident with probability 20%, and a low-risk client has an accident with probability 10%. Accidents in different years occur independently. Given that a randomly chosen client did not have an accident for two years in a row, what is the probability that this is a high-risk client?

10. (10 points) For a random variable X with density

$$p(x) = \begin{cases} cx^{-3}, & x \geq 2; \\ 0, & \text{otherwise}, \end{cases}$$

find c and find $\mathbf{E}X$.

Cumulative Probabilities of the Standard Normal Distribution

The table gives the probabilities $\alpha = \Phi(z)$ to the left of given z–values for the standard normal distribution.

For example, the probability that a standard normal random variable Z is less than 1.53 is found at the intersection of the 1.5 rows and the 0.03 column, thus $\Phi(1.53) = P(Z \leq 1.53) = 0.9370$. Due to symmetry it holds $\Phi(-z) = 1 - \Phi(z)$ for all z.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Quantiles of the Standard Normal Distribution

For selected probabilities α, the table shows the values of the quantiles z_α such that $\Phi(z_\alpha) = P(Z \leq z_\alpha) = \alpha$, where Z is a standard normal random variable.

The quantiles satisfy the relation $z_{1-\alpha} = -z_\alpha$.

α	0.9	0.95	0.975	0.99	0.995	0.999
z_α	1.282	1.645	1.960	2.326	2.576	3.090