University of Nevada, Reno
STAT 461 Midterm 1
October 5, 2026

Name _____

Your exam should consist of 5 problems

Explain your answers, please

You can use a two-sided handwritten note and a calculator

Answer can be in decimal or exact form

Problem	Points	S C O R E
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Consider a committee of 5 people, including the President and the Secretary. You need to select them out of 10 people. How many ways to do this?

Solution: *Way 1.* First, select the President in 10 ways. Then, select the Secretary in remaining 9 ways, then select 3 remaining committee members out of remaining 8 people. For that last choice, order does not matter, and this is equivalent to selecting a subset. In total, we get

$$10 \times 9 \times \binom{8}{3} = 90 \times \frac{8!}{5!3!} = 90 \times \frac{8 \cdot 7 \cdot 6}{6} = 90 \times 56 = \boxed{5040}$$

Way 2. First, select 5 out of 10 people, no matter the role. This can be done in

$$\binom{10}{5} = \frac{10!}{5!5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

ways. Then out of these 5 people, we select the President in 5 ways, and among the remaining, we select the Secretary in 4 ways. The total number of ways is

$$\cdot 5 \cdot 4 \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 10 \cdot 9 \cdot 8 \cdot 7 = \boxed{5040}$$

2. (10 points) An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are below. Given that a driver has been involved in at least one collision in the past year, calculate the probability that the driver is a young adult driver.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	15%
Young adult	16%	8%
Midlife	45%	4%
Senior	31%	5%

Solution: By the total probability formula, the probability of at least one collision is

$$8\% \cdot 15\% + 16\% \cdot 8\% + 45\% \cdot 4\% + 31\% \cdot 5\% = 0.0583.$$

By Bayes' formula, the probability of the driver being a young adult given at least one collision is

$$\frac{16\% \cdot 8\%}{0.0583} = \boxed{0.22}$$

3. (10 points) A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 1% of the time when the disease is not actually present. Two percent of the population actually has the disease. Calculate the probability that a person actually has the disease given that the test indicates the presence of the disease.

   Solution: The prior probabilities of presence or absence of this disease are 2% and 98%, respectively. By Bayes' formula, we get:

$$\frac{0.95 \cdot 2\%}{0.95 \cdot 2\% + 0.01 \cdot 98\%} = \boxed{0.66}$$

4. (10 points) A survey of a group's viewing habits over the last year revealed the following information. Calculate the percentage of the group that watched **exactly one** of the three sports during the last year

   - 28% watched gymnastics
   - 29% watched baseball
   - 19% watched soccer
   - 14% watched gymnastics and baseball
   - 12% watched baseball and soccer
   - 10% watched gymnastics and soccer
   - 8% watched all three sports.

   Solution: *Way 1.* The percentage of people who watched at least one sport can be found using the classic inclusion-exclusion formula:

   $$28\% + 29\% + 19\% - 14\% - 12\% - 10\% + 8\% = 48\%.$$

   From this, we subtract each probability of the double intersection and add back twice the probability of the triple intersection. This way, we count each double intersection zero times and the triple intersection $1 - 1 - 1 - 1 + 2 = 0$ times. The result is

   $$48\% - 14\% - 12\% - 10\% + 2 \cdot 8\% = \boxed{28\%}$$

   Way 2. First, compute the probabilities that two but not three sports are watched. Gymnastics and baseball but not soccer: $14\% - 8\% = 6\%$; Baseball and soccer but not gymnastics: $12\% - 8\% = 4\%$; Gymnastics and soccer but not baseball: $10\% - 8\% = 2\%$. Then the share of people who watched:

   - Gymnastics only: $28\% - 6\% - 2\% - 8\% = 12\%$
   - Baseball only: $29\% - 14\% - 12\% + 8\% = 11\%$
   - Soccer only: $19\% - 12\% - 10\% + 8\% = 5\%$

   Summing them, we get the answer $12\% + 11\% + 5\% = \boxed{28\%}$

5. (10 points) An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto policy and a homeowners policy will renew at least one of those policies next year. Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto policy and a homeowners policy. Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

   Solution: This time, we still need to apply Bayes' formula, but to do it for auto, home, and both would be a mistake, since these are not disjoint and do not form a partition. However, we can consider three cases:

   - The percentage of people who get auto but not home insurance is $65\% - 15\% = 50\%$
   - The percentage of people who get home but not auto insurance is $50\% - 15\% = 35\%$
   - The percentage of people with both types of insurance is 15%

   The percentage of people who have auto or home insurance policy is $65\% + 50\% - 15\% = 100\%$ by the classic inclusion-exclusion formula for two events. Therefore, we might as well, without loss of generality, consider that the entire sample space consists of people who have at least one type of insurance. Thus, the three events above (auto not home; home not auto; auto and home) form a partition of this sample space. Applying the total probability formula for this partition, we get that the probability of renewal is

$$40\% \cdot 50\% + 60\% \cdot 35\% + 80\% \cdot 15\% = \boxed{53\%}$$