STAT 758 Final Exam

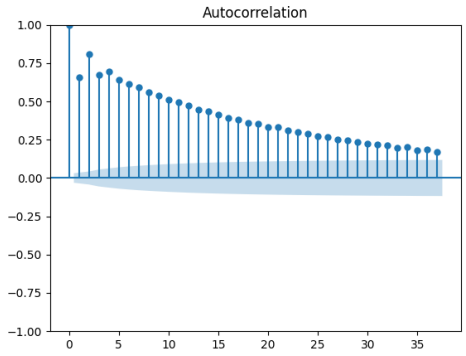
December 14, 2022

Name _____

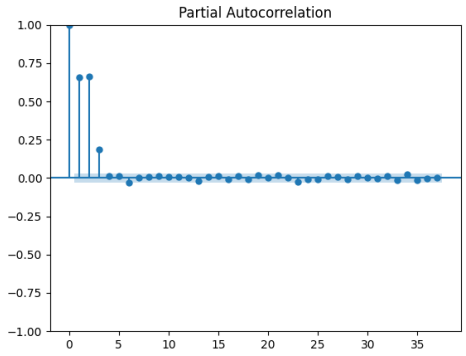
Your exam should consist of this title sheet, followed by 8 problems. The total score is 100.

Problem	Points	S C O R E
1	15	
2	15	
3	10	
4	10	
5	45	
6	10	
7	10	
8	10	
Total:	125	

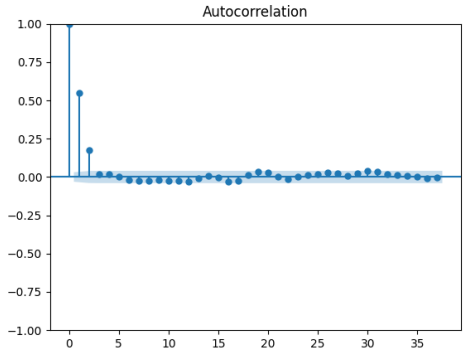
1. (15 points) Classify time series A, B, C using ARMA models.



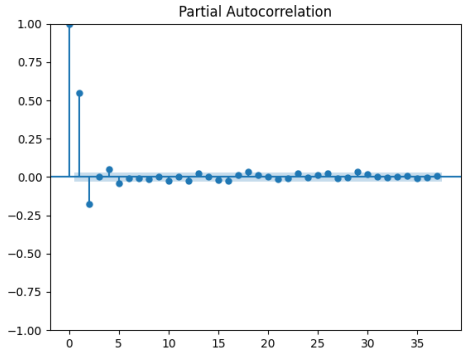
(a) Series A



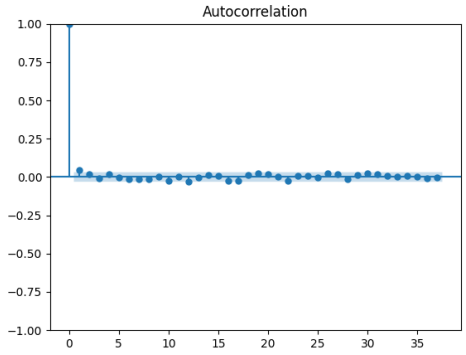
(b) Series A



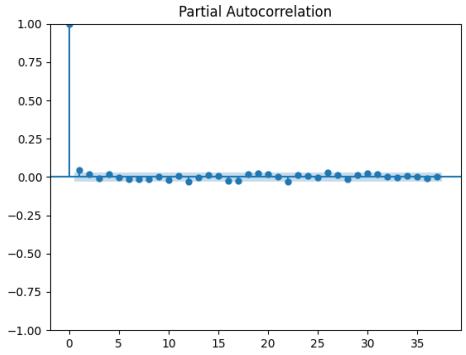
(c) Series B



(d) Series B



(e) Series C



(f) Series C

2. (15 points) Give an example of a sequence which is weak but not strong white noise.

3. (10 points) Classify the model

$$X_t = 1.3X_{t-1} - 0.3X_{t-2} + Z_t - 0.3Z_{t-1} - 1.5.$$

4. (10 points) Consider the MA(2) model

$$X_t = Z_t + aZ_{t-1} + bZ_{t-2} + c,$$

with $a = 0.5$, $b = 0.2$, and $c = 0.6$. Assume Z_t are i.i.d. with mean zero and standard deviation σ. Find its ACF for all lags.

5. (45 points) For the model from the previous problem, find the CLT for the estimates $\hat{a}$, $\hat{b}$, $\hat{c}$ for the estimates of a, b, c.

6. (10 points) Consider the stochastic volatility model

$$X_t = 1.2 + V_t Z_t,\ V_t^2 - 2.3 = 0.3(V_{t-1}^2 - 2.3) + W_t.$$

Here, Z_t and W_t are independent and Z_t are identically distributed with mean zero and variance 2.5, while W_t are identically distributed with mean zero and variance 4. Explain why this model is stationary. Find the long-term mean and variance for X_t.

7. (10 points) For a Brownian motion X with drift 3, find the conditional density of $X(7)$ given $X(2.5) = 1.5$.

8. (10 points) Write the general form of the $\text{SARIMA}(1,0,0) \otimes (1,0,1)_4$.