STAT 758 Final Exam
December 14, 2020

Name _____ Student ID #_____

Problem	Points	S C O R E
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) For an autoregression of order 1:

$$X_t = 1 - 0.2X_{t-1} + Z_t$$

where Z_t is white noise with $\mathbb{E}[Z_t^2] = 4$, find the expectation and variance of X_3 given $X_0 = 0$.

2. (10 points) Consider the autoregression of order 2:

$$X_t = 0.2X_{t-1} - X_{t-2} + Z_t,$$

with white noise Z_t. Find whether it is causal.

3. (10 points) Find the a for which the following time series has unit root:

$$X_t = 2X_{t-1} - aX_{t-2} + Z_t - 0.5Z_{t-1}.$$

Classify this model in this case as ARIMA.

4. (10 points) Find the values of b for which the model is invertible:

$$X_t = 0.2X_{t-1} + Z_t + bZ_{t-1}.$$

5. (10 points) In the ARMA(1, 1) model, express X_t in terms of $Z_t, Z_{t-1}, \ldots$

$$X_t = -2 + 0.4X_{t-1} + Z_t + 0.5Z_{t-1}.$$

6. (10 points) Invert the model in the previous problem: Express Z_t in terms of $X_t, X_{t-1}, \ldots$

7. (10 points) For the same model, compute the autocovariance function.

8. (10 points) For an AR(2) model, assume you have estimates $\hat{\beta}_1 = 0.2$ and $\hat{\beta}_2 = -0.01$ for $T = 400$ data points. Find the 99% approximate confidence intervals for β_1 and β_2.

9. (10 points) For the previous problem, assume you have $\overline{x} = -2.5$ and $s^2 = 7.29$ (variance of the white noise). Find the approximate confidence interval for μ, the mean of the time series.

10. (10 points) For an MA(1) model, assume you have $\hat{\alpha}_1 = -0.5$ for $T = 400$ data points. Find the 95% approximate confidence interval for α_1. Do you reject the hypothesis that the model is invertible?